The Simulation of Strong Gravitational Lensing of Type Ia Supernovae.

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Abstract

Observing gravitationally lensed supernovae, or supernovae in general is a tricky task, it is very difficult and often impossible to get advanced warning of when a Supernova (SN) will occur, and they take place over very short periods of time. The aims of this project were to create a simulation demonstrating the evolution of a Type Ia SN as it is being strongly gravitationally lensed. In addition to this, MCMC methods were used to look for potential degeneracies between parameters that may or may not affect the appearance of a supernova, in this case the source supernova light. The aims of this project were achieved; the supernova lensing system was successfully reconstructed, and the finding of this research suggests that there is no degeneracy between the supernova light and other parameters in the lens model.

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1 Introduction

1.1 Cosmology

Cosmology is a branch of science concerned with the origin, evolution, and future of the universe. Cosmology ties together many disciplines of physics such as particle physics, general relativity, and quantum mechanics. Of the many great unknown questions in science, two of them (at the very least) are in the field of cosmology; what are dark matter and dark energy, and how exactly do they behave?

Dark matter is a hypothetical form of non-baryonic matter that is thought to account for roughly 85% of matter in the universe [1]. Some of the first radio astronomy observations showed that galaxies spun much faster than expected in their outer regions based on the mass of observed matter contained in the galaxy [2]. This could only be the case if the galaxies were significantly more massive that previously thought. It was then that dark matter was first proposed to explain the behaviour of the observed galaxies [3].

Dark energy is a hypothetical form of energy that is driving the accelerated expansion of the universe. Because it is commonly believed that all Type Ia SN go supernova at the same luminosity, or are at least standardisable, they are very useful for measuring distances in the universe, and hence the universe's expansion rate . When astronomers were observing Type Ia SN, they saw that many were dimmer than expected based on their age, even with an understanding that the universe was expanding [4]. Many investigations have been conducted to probe how reliable using Type Ia SN are to get a value for the expansion rate of the universe, or if they can be accurately used at all [5].

Our most successful theory of gravity is General Relativity (GR). In space-times governed by GR, we find that the Universe must be contracting or expanding [6]. Observations tell us it's expanding; this led to the realisation that the Universe must have begun from a very small point. Further observations of the accelerating expansion, structure, growth, and galaxy behaviour led to the construction of the LCDM model, now considered the standard model of cosmology.

The ΛCDM model is a model that takes into account dark energy and dark matter, it provides an explanation of many properties of the universe, including the Universe's accelerating expansion. The model is a parameterisation of the Big Bang cosmological model in which the universe contains three major components: a cosmological constant associated with dark energy, cold dark matter, and third, ordinary matter. The ΛCDM model explains the existence and structure of the cosmic microwave background, the large-scale structure in the distribution of galaxies, and the accelerating expansion of the universe observed in the light from distant galaxies and supernovae [7]. By understanding gravitational lenses and Type Ia SN, we can learn more about the properties of dark matter and dark energy. Therefore, the aim of this project was to investigate how a Type Ia SN could be modelled using the gravitational lensing software 'Lenstronomy'.

1.2 Gravitational Lensing

A gravitational lens is a massive celestial body, such as a galaxy or a cluster of galaxies, between a distant light source and an observer that is capable of bending the light from the source as the light travels toward the observer. We often see multiple or magnified images as a result of the lensing.

There are three main types of gravitational lensing:

- Strong Lensing: The lens is a large mass and the deflection is comparatively large. The alignment between the observer, source, and lens is favourable, so an observer sees two or more separate images of the source.
- Weak lensing: This is a small effect that can only be observed by looking at the shapes of numerous galaxies that show preferential alignment in one direction, indicating lensing. There is no single

source/lens, but many background galaxies acting as sources and many foreground galaxies acting as lenses.

- Microlensing: The lens is usually a small mass like a star. The mass is small so the Einstein radius is correspondingly small, meaning we do not see multiple images. Instead of seeing multiple images, the image of the source appears to brighten for the duration of the lensing event; for example, a star will brighten if a sufficiently massive planet transits it.

In this report, the focus will be on strong gravitational lensing.

Gravitational lensing allows us to probe the distribution of matter in galaxies and clusters of galaxies, and enables observations of the distant universe.

Strong gravitational lensing helps us understand dark matter. Although dark matter is not visible, astronomers can detect its influence by observing how the gravity of massive galaxy clusters, which contain dark matter, distorts the light of more-distant galaxies located behind the cluster.

By observing the areas around galaxy clusters, astronomers we can identify distorted background galaxies and examine their distortions to reveal where the densest concentrations of matter lie. Mathematical models of these results shed light on the location and properties of the lensing material, both visible and "dark".

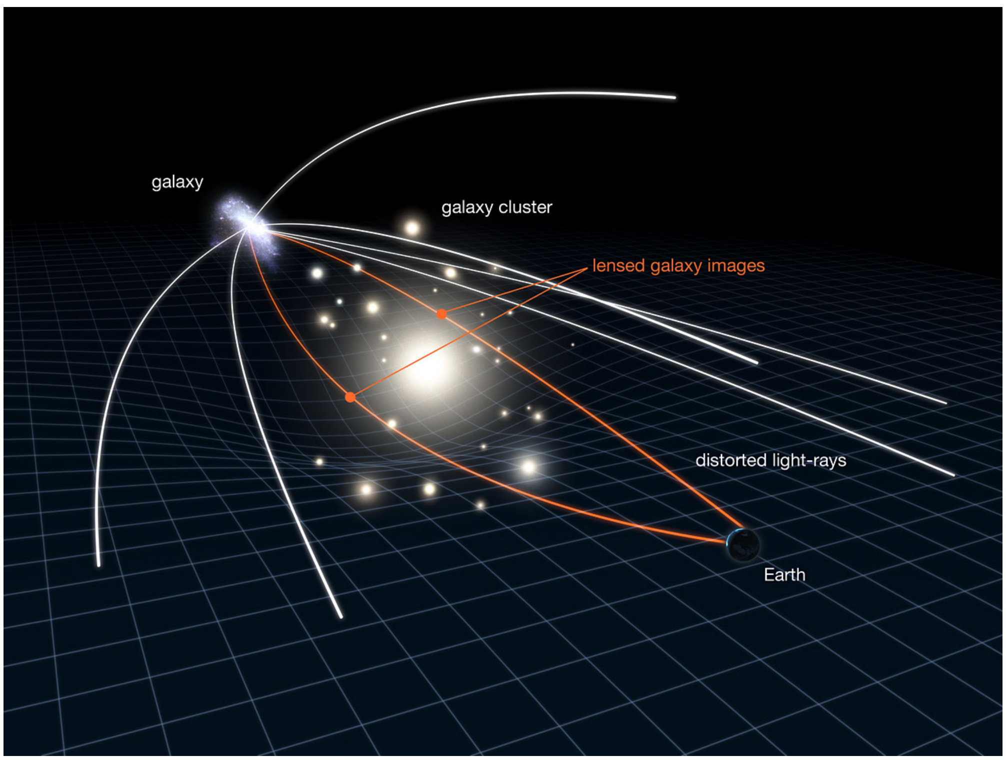

Figure 1: Diagram of strong gravitational lensing. ESA/Hubble

An Einstein ring is created when light from a distant object such as a galaxy, passes by a massive object en route to the Earth. Due to gravitational lensing, the light is bent making it seem to come from different places. If the source, lens, and observer are all in perfect alignment, the light appears as a ring, an Einstein ring. This expression for the Einstein radius is specifically for the EPL model.

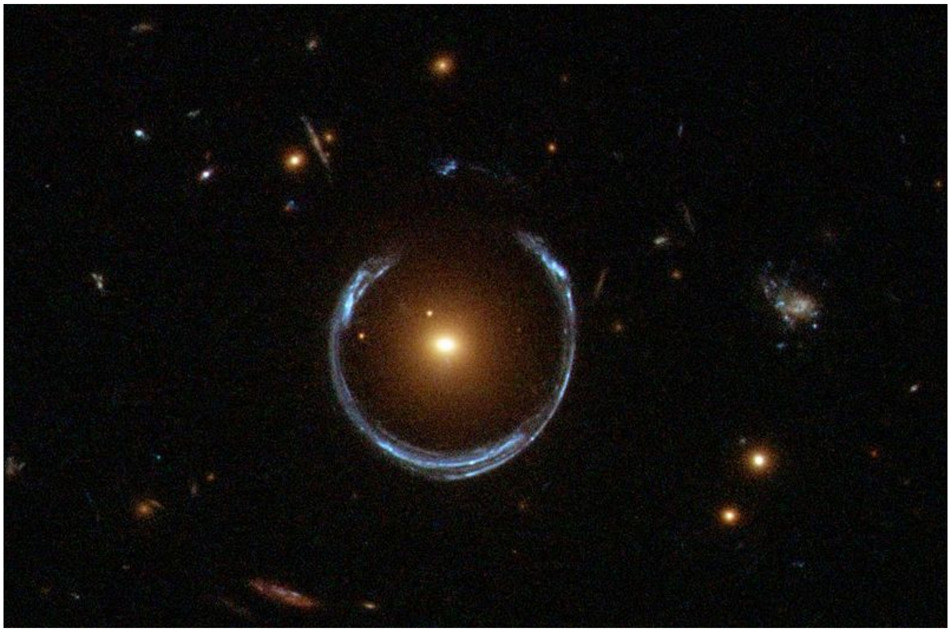

Figure 2: Example of Einstein ring. ESA/Hubble & NASA derivative work

1.3 Type Ia Supernovae

Type Ia supernova are a type of supernova that occur in binary systems where one of the stars is a white dwarf. The companion star can be a a main sequence star or another white dwarf [8, 9]. They are thought to be the result of the explosion of a white dwarf in a binary system as it goes over the Chandrasekhar limit due to accretion from the companion star.

One of the most effective ways to measure distances to distant galaxies tens to hundreds of of parsecs (pc) away is to use the luminosity of Type Ia SN. It is thought that Type Ia SN produce a consistent peak luminosity because of the fixed critical mass at which a white dwarf will explode. So it is thought that any supernovae of this type observed in a very distant galaxy should also have the same luminosity as a Type Ia SN in any other galaxy, only its apparent magnitude would be different.

Their consistent peak luminosity allows their explosions to be used as ”standardisable candles” to measure the distance to their host galaxies. The visual magnitude of a type Ia supernova, as observed from Earth, indicates its distance from Earth. However it is currently understood that Type Ia SN luminosity is dependant on the environment that it is in, their peak luminosity depends on the age of the stellar progenitor systems, where more luminous Type Ia SN appear in younger stellar populations. This means that it's not so simple to assume that the peak luminosity of Type Ia SN will be the same everywhere in the universe, hence why their luminosities must be standardised [10, 11].

How do Type Ia SN inform us about the expansion of the universe according to the ΛCDM model?

Compared to nearby SN, distant SN appear much too dim, even for a universe which has been expanding (constantly) over the last several billion years. This implies that the distances to the SN are larger than expected, which means that the distances to the SN must have increased over and above what they would have been if the rate of expansion did not accelerate with time. This is only possible by the effect of additional acceleration, i.e., the rate of expansion of the Universe increases with time. This is an observation of dark energy at work.

This project will be exploring the use of strong gravitational lensing to observe how the appearance of a lensed SN changes with time.

It is worth mentioning that there are some cosmological models that do not rely on Type Ia SN to measure distances or the expansion rate of the universe; however, Type Ia SN could be used provided the model was well understood and a correction for these effects could be applied. The strength of gravity can vary with time or space in some modified gravity models. This means that if that the strength of gravity varies over cosmic time and space, some key properties of the white dwarf progenitors of the Type Ia SN, such as their mass, can become red-shift dependent [12].

The red-shift dependence of these key properties may in turn affect the intrinsic peak luminosities of the Type Ia SN. If this is the case, then the measurement of the acceleration of the universe that such theories introduce to explain the observed SN may need to be reinterpreted as the very result of their introduction. In the extreme case, modified gravity theories may produce an acceleration that is no longer supported by the Type Ia SN data once the data is reinterpreted in the context of the new theory [12].

The peak luminosity of the SN is proportional to the amount of Nickel formed in the SN, which is a fixed fraction of the Chandrasehkar mass, which is proportional to G. The Chandrasehkar mass is the mass at which the electron degeneracy pressure that supports the White Dwarf against its own gravity is overcome. Because the Chandrasekhar mass is proportional to the strength of gravity, if we have a theory where gravity is stronger than normal, less mass is required to overcome this pressure. In turn, the amount of nickel the SN produces is related to the Chandrasekhar mass, and the amount of nickel plays a role in the peak luminosity of the SN [13]. However in this project we will be working within the ΛCDM model, meaning we do not need to consider these effects.

2 Methods for Generating Lens Images

2.1 Software to generate lens simulation

The modelling of gravitational lensing of SN for this project is done using a multi-purpose gravitational lens modelling software package known as 'Lenstronomy' [14].

Lenstronomy is able to reconstruct the lens mass and surface brightness distributions of strong lensing systems. Lenstronomy also supports a wide range of analytic lens and light models in arbitrary combinations. Lenstronomy can make several measurements including deriving constraints on dark matter properties in strong lenses, measuring the expansion history of the universe with time-delay cosmography, measuring cosmic shear with Einstein rings and decomposing quasar and host galaxy light.

Many of the measurement tools Lenstronomy provides have been used in the three Python scripts that were written. The first of the Python scripts was written to model the evolution of the lens system and generating simulated images, the other to fit the images using an MCMC, and the final one which was almost identical to the one that fit the images using an MCMC, but it was used to check for potential parameter degeneracies in the lens system as discussed in (4.2).

2.2 Parameters in lens and light model

There are various parameters required in order to explain the appearance and behaviour of a lensing system, some make more of a difference than others, however, all are necessary. Here are the various profiles used in the full model for our lensing system.

There are two lens model profiles in use that describe the mass of the objects causing the lensing. The Elliptical Power Law (EPL) profile gives the mass distribution of an elliptical galaxy (our gravitational lens), then the shear which explains how the gravitational lens is distorted by objects that aren't the dominant gravitational lens. There are multiple options lenstronomy provides for various galaxy shapes, but this was chosen because it represents our galaxy the best.

We use an elliptical Sersic profile to describe the lens light. The profile in lenstronomy is called sersic_ellipse. It is a mathematical function that tells us how the intensity of an elliptical galaxy varies with distance from its centre.

$$I(r) = I(0)exp^{-b_n \frac{r}{r_e} \frac{1}{n}} \tag{1}$$

Where $I(0)$ is the central intensity, and r_e is a scale radius. The quantity b_n is a function of the shape parameter n, and is chosen so that the scale radius encloses half of the total luminosity i.e. the half-light radius [15].

Lastly we have the two source light model profiles that describe the light coming from the source galaxy and from the supernova. We have the same sersic_ellipse profile that tells us how the intensity of an elliptical galaxy varies with distance from it's center. Then the Gaussian profile which determines how the intensity of light from our supernova appears.

Here are the parameters for the lens mass for the EPL:

- Einstein Radius (θ_E): This is the radius of the Einstein ring in arc seconds.

$$\theta_E = \frac{M\theta^{(\gamma-3)}}{(\pi d_{ol}^2)^{(1/\gamma-1)}\Sigma_{\text{crit}}}, \tag{2}$$

  where M is the lensing galaxy's mass in solar masses, θ is the angular size of the lens galaxy, γ is the slope of the power law in the mass profile, d_{ol} is the comoving distance between the observer and the lens galaxy (in kpc), and Σ_{crit} is the critical surface density of the lens galaxy. Using this equation and finding values for the respective parameters, we can find the Einstein radius of a physically realistic galaxy.

- Ellipticity parameters of lens: Determines how elliptical the lensing mass is.

$$e_1 = \frac{1 - q_m}{(1 + q_m) \cdot \cos(2\phi_m)} \tag{3}$$

$$e_2 = \frac{1 - q_m}{(1 + q_m) \cdot \sin(2\phi_m)} \tag{4}$$

  where q is the aspect ratio of the lens galaxy, and ϕ is the direction of the ellipticity; i.e. how much the circle squashed along the x axis, y axis.

- Gamma (the Power law slope): The slope of the Power Law describing the surface density [16]:

$$k(r) = \frac{2-t}{2}\left(\frac{b}{r}\right)^t \tag{5}$$

  The power law slope comes from this equation Where 0 < t < 2 is the slope of the profile, b > 0 is the scale length and r > 0 is the distance from the centre of the mass distribution. Such a profile arises from a spherically symmetric three-dimensional mass distribution.

In order to turn the profile into an elliptical surface mass distribution, the x-axis is now stretched by a factor of q - 1, where $0 < q \leq 1$ is a constant parameter of the new profile. It is clear that after this operation, the formerly circular isodensity contours r = const. have indeed become ellipses with semi-major axis r/q, semi-minor axis r, and axis ratio q. The resulting elliptical power law profile can be written as:

$$k(R) = \frac{2-t}{2}\left(\frac{b}{R}\right)^t \tag{6}$$

where R is the elliptical radius defined by:

$$R = \sqrt{q^2x^2 + y^2} \tag{7}$$

- center_x, center_y: Determines the alignment of the observer and lens; the x and y parameters are the position of the centre of the profile in arcsec.

For the external shear:

- gamma1, gamma2: Two components that describe the distortion of the lensed objects in the x- and y-direction by external objects that do not form part of the primary lens.

Here are the parameters for the lens light. With the sersic_ellipse profile.

- Magnitude: Describes the luminosity of the lens light in apparent magnitude
- R_sersic: The half light radius of the lens.
- n_sersic: Controls the degree of curvature of the profile. The smaller the value of n, the less centrally concentrated the profile is.
- Ellipticity parameters of the lens (e_1 and e_2): Describe the elliptical profile of the light given that we have an elliptical galaxy.
- center_x, center_y: Controls for alignment of observer and lens, in this case, how offset is the lens from the observer. The x and y parameters are the position of the centre of the profile in arcsec.

Here are the parameters for the source light (galaxy and supernova). For galaxy we again have the sersic_ellipse profile.

- magnitude: Describes the luminosity of the source galaxy light in apparent magnitude
- R_sersic: Half light radius of the source galaxy.
- n_sersic: Controls the degree of curvature of the profile. The smaller the value of n, the less centrally concentrated the profile is.
- Ellipticity Parameters of the source (e_1 and e_2): describe the elliptical profile of the light given that we have an elliptical galaxy.
- center_x, center_y: Controls for alignment of source and lens, in this case, how offset is the source from the lens. The x and y parameters are the position of the centre of the profile in arcsec.

Here are the the supernova parameters, it is not elliptical so there are no e_1 or e_2 parameters.

- magnitude: Describes the luminosity of the source galaxy light in apparent magnitude.
- sigma: Standard deviation of the Gaussian function which describes the luminosity of the supernova.
- center_x, center_y: Controls for alignment of source and lens, in this case, how offset is the source from the lens. The x and y parameters are the position of the centre of the profile in arcsec.

2.3 Gathering Parameter Values

2.3.1 Distances of objects

The first thing to do was to set the distances of the lens, source, and observer from each other. The following distances were chosen: from observer to lens, 1000 Mpc; from lens to source, 500 Mpc; from observer to source, 1500 Mpc. These distances inform the magnitudes of the supernova and galaxy that will be observed. These distances can be set to whatever one thinks is reasonable.

2.3.2 Magnitude of lensed supernova

The average absolute magnitude of a Type Ia SN is -19.5 at its peak [17]. In the code, the magnitudes are in apparent magnitudes, so the absolute magnitudes were converted in order for them to be correct in the code.

This is the equation required to do so, the distance modulus, and the values were calculated by hand (the distance must be in parsecs):

$$m - M = -5 + 5\log(d) \tag{8}$$

m is the apparent magnitude, M is absolute magnitude, and d is the distance from the observer.

After inputting the distance of the observer to source (1500Mpc), and the absolute magnitude (-19.5) we get a value for the apparent magnitude of roughly 21:

$$m - (-19.5) = -5 + 5\log(1.5 \times 10^9), \tag{9}$$

$$m = -5 + 5\log(1.5 \times 10^9) - 19.5, \tag{10}$$

$$= 21.38 \approx 21 \tag{11}$$

There are three broad stages of Type Ia SN luminosity evolution: first, the rapid increase in luminosity that lasts for 5 - 10 days; then, a brief plateau stage that follows which also lasts from 5 - 10 days where the brightness is relatively constant; and lastly, a stage of gradual declining luminosity to its "before supernova" stage which may last around 30 - 40 days [18, 19]. It is often stated that Type Ia SN have the same luminosity at their peaks, with the only difference in their apparent magnitudes, although as discussed in (1.3) that is not entirely accurate.

For the sake of simplicity in the simulation, the phase of increasing luminosity will last 10 days; the plateau stage will last 10 days; and the decrease in luminosity will last 40 days. Since we know the change in supernova luminosity against time now, from before and after the peak, the starting luminosity can be easily determined.

Since the luminosity of the supernova increases (meaning the magnitude decreases) by $\Delta 0.7$ per day before it reaches the peak, 10 days before the peak, the luminosity will be ($10 \times 0.7 =$) 7 lower. So the initial magnitude of the supernova will be ($21 + 7 =$) 28.

The magnitude decreases back to its original luminosity in 40 days, so for it to decrease from the peak of 21, to the initial magnitude of 28 in 40 days, the magnitude must decrease by ($7/40 =$) 0.175 per day.

2.3.3 Magnitude of lensed galaxy

Equation (8) can also be used for the luminosity calculation of galaxies. With an absolute magnitude of -23, we get an apparent magnitude of 17, which is reasonable for the distance, especially because our lensing galaxy is elliptical [20].

Again, the distance of the galaxy was inputted, and the luminosity of a typical elliptical galaxy was used as the basis for the luminosity of this galaxy [20]:

$$m - (-23) = -5 + 5\log(1 \times 10^9), \tag{12}$$

$$m = -5 + 5\log(1 \times 10^9) - 23, \tag{13}$$

$$= 17 \tag{14}$$

2.3.4 Mass of lensing galaxy

A relatively small elliptical galaxy of 9.7×10^{10} times the mass of the Sun was used to lens the supernova. This a reasonable mass for an elliptical galaxy [21].

2.4 Implementation Into Code

Using this information found from the literature, it was time to implement this information into the code. The respective mathematical equations (2), (3), and (4) were added to the lenstronomy file in Python form so that we have the necessary variables for the simulation. As were the distances, mass, and the other parameters.

Lenstronomy has dictionaries where parameters that will decide the behaviour and appearance of our lens, galaxy, and SN are stored. So the required variables where declared and stored in the respective dictionaries, which contain their keyword arguments.

One thing hasn't been mentioned so far, and that's the instrument noise, it's entirely unrealistic to produce an image as if there were no noise, so for the purposes of this simulation, simulated noise from the Hubble Space Telescope WFC3 F160W instrument was used.

In order to keep a record of the latest result from a run that the program had completed, there was the added functionality of saving the latest modelled lens image to a folder. An image could be saved manually too, that is useful if a comparison needed to be made between the latest image and previous images.

2.5 Preliminary Results and Discussion

In Figure 3 we see the first images we have with all the preliminary parameters and noise chosen and correctly applied to the lens simulation. The image on the right is with noise applied which is what we would see in a non-processed image. The image on the left is with no noise. It is important to note that this is when the supernova is at arbitrary brightness, a value was chosen that would just be easy to observe. There is currently no external shear, hence the circular shape of the lens. The model was run with preliminary values to ensure that the code was working in terms of noise generation, removing errors, and actually generating a lens model at all.

This tells us that the lens is constructed as we would expect, and it gives a guide for what we should expect from the images before and after the supernova occurs. Mainly that the lensed object (SN and galaxy) will be less visible because of the reduced luminosity of the supernova.

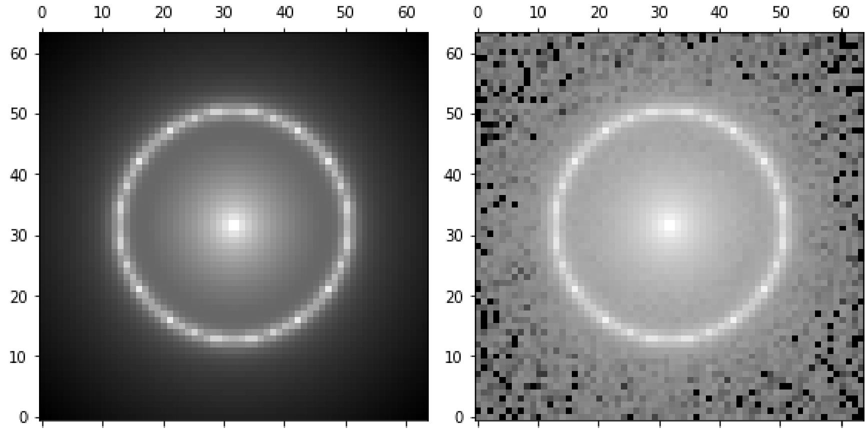

Figure 3: Preliminary lensing image with the numbers denoting the width and height dimensions of the images in pixels.

2.6 Further implementation

To make the image appear more realistic, shear was added, as was a displacement in the x- and y-position of the lens, galaxy, and SN; this is because it's very rare to get a perfect alignment between source and lens and an image without shear; and because the purpose of this report is to simulate what one may actually observe, it would serve no real purpose to have such a simulation. The "true" luminosity values were also added as discussed in (2.3.2).

So after having adding in the actual parameter values that were expected, we get a more realistic image based on what we would actually observe. The difference in colour and intensity of the light is due to Python plotting functions. But now we see the distortion of the lens as desired. This singular image is the supernova at peak brightness.

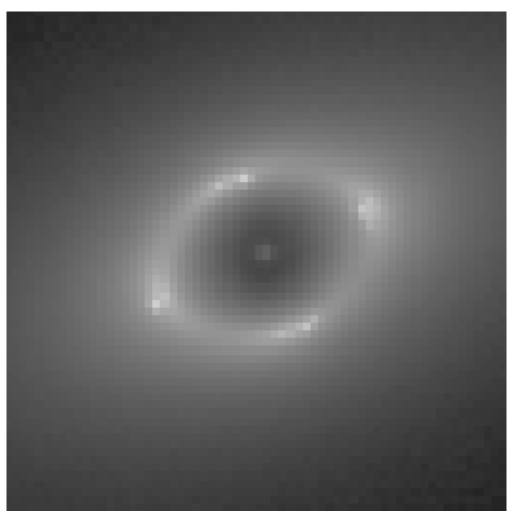

Figure 4: Supernova at peak luminosity with the correct parameters as discussed in (2.3.2) applied to the image.

The next thing to do is to generate multiple images, here we apply of knowledge our supernova evolution discussed in (2.3.2). Since we now know the rate of increase and decrease in SN luminosity, this will be displayed in the code. However, because over many of the days there is little perceptible change in the luminosity of the SN, not all 60+ days of the SN evolution were plotted, only a few key periods, these being the incremental increase in luminosity, the plateau period, then the incremental decrease in luminosity.

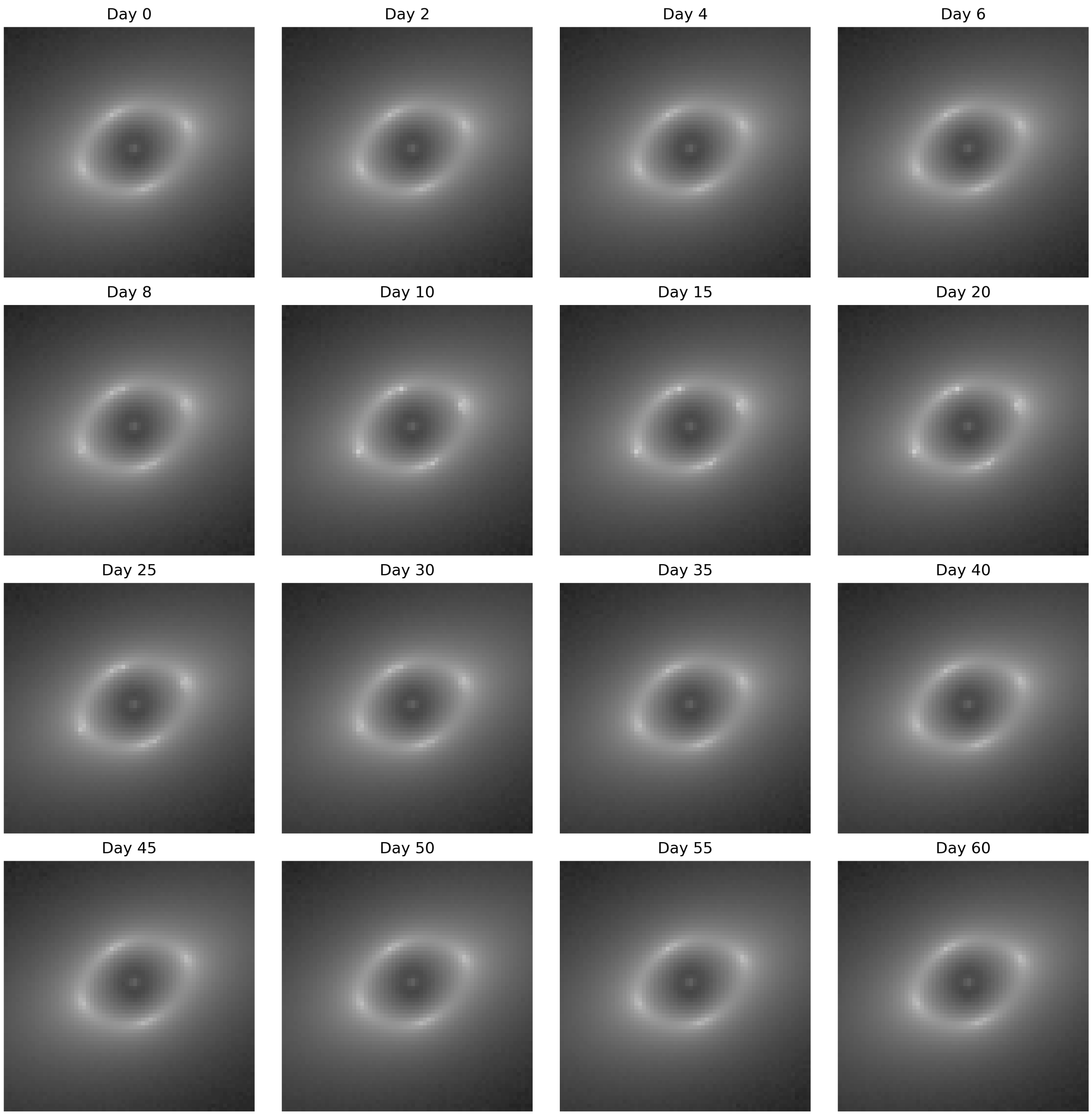

Figure 5: The evolution of the strongly lensed supernova over a period of two months of increasing, static, and decreasing luminosity.

2.7 Final results

In Figure 5 we now see multiple image at selected periods of the increase, plateau, and decrease in luminosity of the supernovae. For most of the images it is difficult to see the SN in the lensed light source from the galaxy. This tells us something important about the periods that we could expect to observe a lensed SN.

3 Discussion of Findings From Lens Images

From Figure 5, we can see that it's easiest to observe the SN between days 8 to 35. This gives a 27 day window in which it would be easiest so spot the SN, this is when the lensed SN sufficiently outshines its host galaxy to be visually detected.

Depending on alignment of the galaxy (which contains the supernova) and the lensing galaxy, the SN may be more visible on the top, bottom, left, or right of the image. In this simulation, the SN is more visible on the top and bottom.

We know the amount of time that the supernova spends in its respective stages. So if a SN were to be seen at a certain luminosity in relation to its host galaxy, it may be possible to estimate how long ago the SN occurred, or how it has to go until it reaches peak luminosity.

4 MCMC Analysis

In this section, we now want to attempt to fit a model to our simulated data when the SN is at peak luminosity to match what is done in the analysis of real strong lensing images. We also go on to investigate potential degeneracies between different model parameters as discussed in (4.2), we will do this using Monte-Carlo Markov-Chain (MCMC) methods.

Markov-Chain Monte-Carlo methods utilise Bayesian statistics. In Bayesian statistics, probability expresses the chance of an event occurring when prior knowledge of an experiment is explicitly taken into account. This is expressed in Bayes' theorem [22, 23, 24].

$$P(\theta|D) = \frac{P(D|\theta)P(\theta)}{P(D)}, \tag{15}$$

The posterior distribution $P(\theta|D)$ is the probability of measuring a parameter θ given the data D, and is given by the likelihood ($P(D|\theta)$, the probability of measuring the data given the model parameters) multiplied by the prior $P(\theta)$ and normalised by the evidence $P(D)$.

MCMC methods proceed by drawing a sample at random from the posterior distribution and then stepping in a random direction in the parameter space. If the new point is of a higher probability, the step is accepted and added the chain; otherwise, it is rejected. The sampling proceeds in this way until the chain has explored the majority of the posterior and the peak of the probability distribution is found. The MCMC sampler used in this work was the affine-invariant ensemble sampler provided in the emcee package [25].

The posterior distribution can be found analytically or via grid methods when the number of parameters is small, but for the majority of cosmological applications this is not the case. We thus turn to sampling methods in order to approximate the posterior distribution. One such sampling method is MCMC.

Why exactly do we want to use MCMC analysis? It's because Markov-Chain Monte-Carlo sampling provides a method of (systematic) random sampling from high-dimensional probability distributions. MCMC methods draw samples where the next sample is dependent on the existing sample, this is the Markov Chain part. This allows the algorithm to narrow in on the quantity that is being approximated from the distribution, even with a large number of random variables [23].

4.1 Image Reconstruction

We need to make an accurate reconstruction of the "observed" image so that we know the MCMC method works. Working means that the parameters converge to the same values as in the lens model generating the image. Sometimes certain parameters can interfere with others causing the reconstructed image to appear incorrect.

There are fixed and free parameters for the lens and the galaxy. The fixed parameters for both were the x- and y-axis position. The rest of the parameters for each were variable. For the lens light, these were the ellipticity parameters, and the R_ and n_sersic parameters. For the galaxy, the variable parameters the same respective parameters as for the lens. Lastly, for the SN, there are no fixed parameters, the variable parameters are the sigma, and x- and y-axis position.

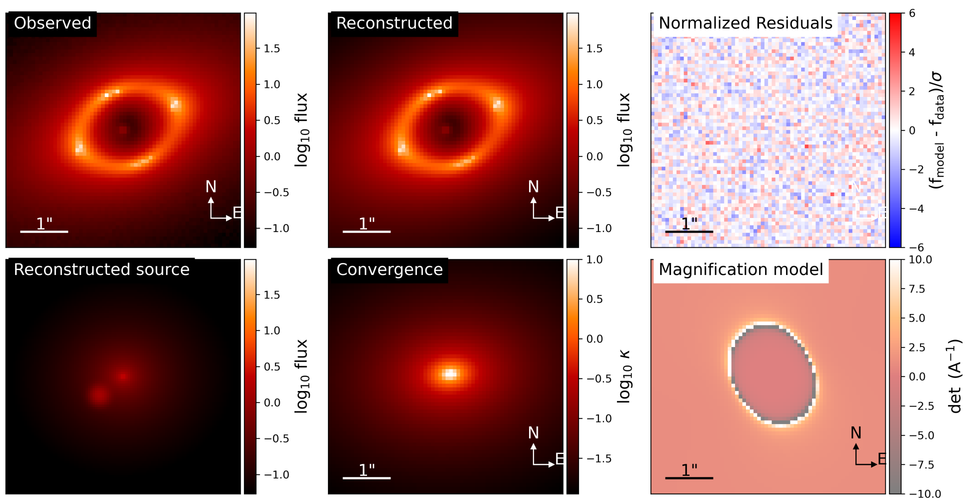

Figure 6: Observed and correctly reconstructed supernova image.

In Figure 6 we see six different plots. The Observed plot is the simulated image of the lensed supernova as seen in Figure 4. The Reconstructed plot is what the MCMC simulation generates. We know that it is correctly reconstructed (other than by eye) because of the plots that display the Normalized Residuals, Reconstructed Source, and Convergence.

The Normalized Residual plot shows the residual difference between the two images. Because we see that it is mainly grey/white, i.e. the residual difference between the two images is close to zero, it means that the image is constructed very well. The Reconstructed Source plot shows that the supernova and galaxy source light were correctly reconstructed, as we will see shortly, if they were not reconstructed correctly, then this would be clearly visible in the residuals plot. The convergence plot is the dimensionless surface density of the lens model, so that plot shows us how the mass of the lens is distributed according to the EPL, and it looks as we expected.

Lastly, the magnification model shows us how magnified the images are. We see that the orientation of the magnification image is not the same as the observed and reconstructed image. The magnification is the inverse of the determinant of the amplification matrix, which contains our convergence and shear terms. so it completely depends on the mass distribution of the lens, and the mass distribution of objects around the lens, (the objects inducing the shear).

One way to tell how well our image is reconstructed is to look at the "chi-square fit" of the reconstructed image. If this value is close to 1.0, this indicates that the reconstruction produced by the MCMC is fit very closely to the observed image. For the image above, we see a chi-squared value of 0.9845713474040759.

4.2 Checking For Degeneracies

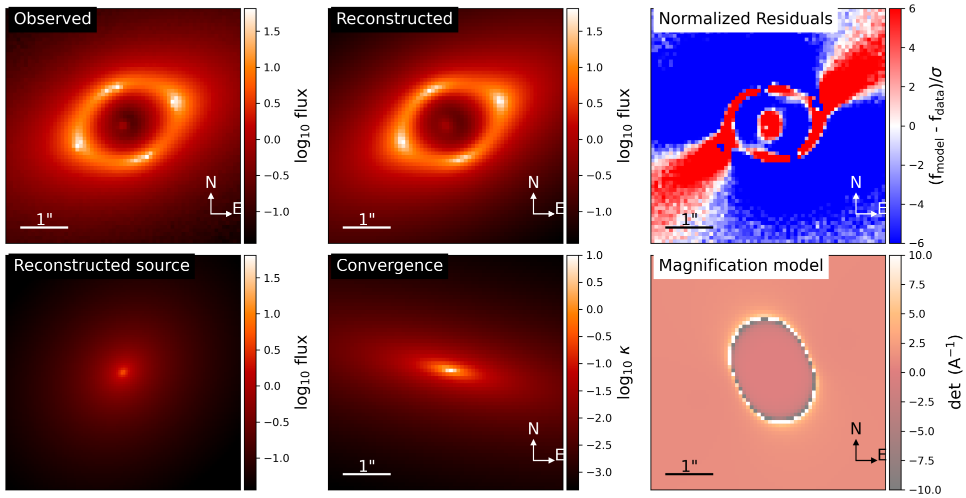

Figure 7: Observed and incorrectly reconstructed supernova image.

In Figure 7 we see a similar image to that of Figure 6, however this is where we remove the supernova light to investigate to what extent removing the source supernova light affects the reconstructed image.

With some models and observations it is possible to get the effect of one parameter confused with another. The shape of a lensed image depends on a few variables such as the alignment of the source and lens, the mass distribution of the lens, and external shear, so when looking at a given image it may not be possible to distinguish which of these quantities is causing the specific distortion. This means that the model parameters describing the lensing system are degenerate, the extent to which they are degenerate depend on things like the model(s) being used for the lens or source for example.

It is important to identify if any parameters may or may not be degenerate, otherwise it makes it tricky to generate physical models if it's unknown which parameters control which observables.

To find out if this was the case, the MCMC was run exactly as normal, but with only one difference: the supernova light was removed. This was done to see what kind of image would be generated if it was removed; maybe other parameters would change to compensate for the missing light and reconstruct the image with remarkable accuracy? If that were to be the case, we would have to investigate what would be causing that.

In Figure 7 we see that the observed plot is the only one that is the same as in Figure 6. Because the source light is not reconstructed since we removed the supernova light, all the other plots will be incorrect. We see that the reconstructed plot does not look like the observed plot, as one might expect since one of the sources of light that would be lensed is no longer there.

The convergence plot is expected given the parameters provided and the exclusion of the SN source light, but it is not what we would want to observe if the image was reconstructed correctly. This indicates that the

MCMC is attempting to compensate for the loss of the SN source light by coming up with an unsual lens model. The magnification plot is also slightly different because lensing of the supernova is not happening. The magnification plot doesn't look drastically different however, because lensing of the source galaxy light is still occurring.

We can have a look at contour plots generated during the run of each of these programs. Here we see a Gaussian type distribution with the values of our parameters. From left to right we have: Einstein radius, gamma, and our ellipticity parameters. This is the plot for the correctly reconstructed image, it is very close to what we expect to see.

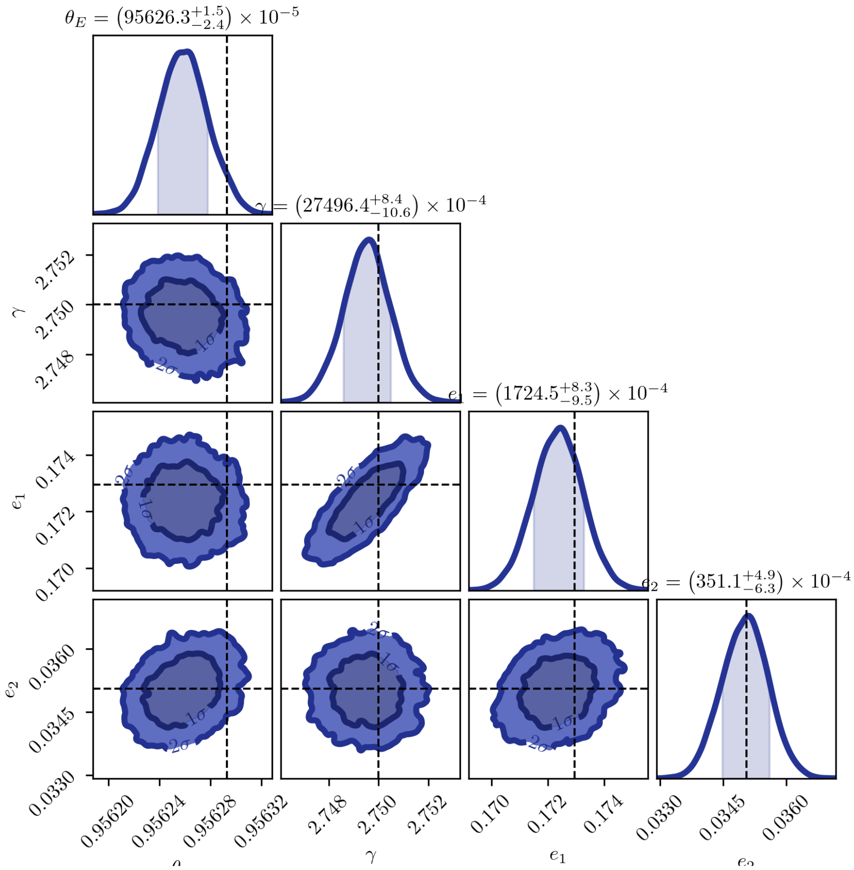

Figure 8: Contour plot of the correctly reconstructed image.

In the below contour plot of the incorrectly reconstructed image, we see that the contours are extremely non-Gaussian, and our displayed values are extremely far from the desired values. This is because the MCMC had difficulties with trying to reconstruct the image without the SN source light.

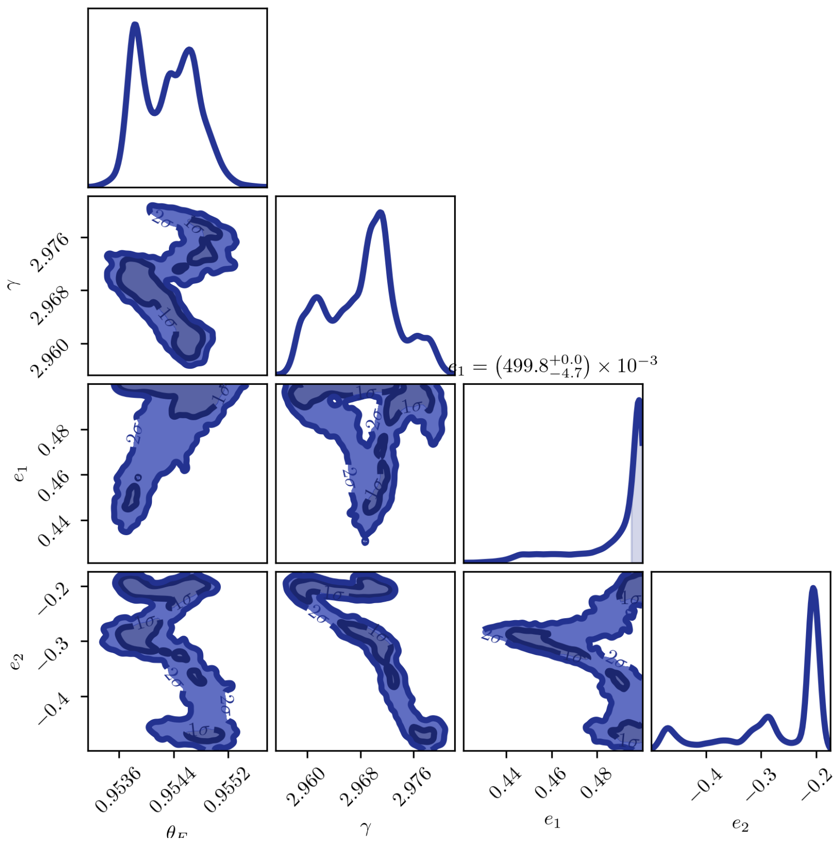

Figure 9: Contour plot of the incorrectly reconstructed image. (The image with no supernova source light).

The most stark difference between Figures 7 and 6 are their residual plots. In figure 7 we see that there is a huge residual difference between the reconstructed and observed image. Removing the supernova light means that the light from the Einstein ring is not as it should be, neither is the radiant light inside and outside the ring. Hence why we see the vivid blue and red denoting that there is a large difference between what we see in the observed image, and what the MCMC tries to construct with incomplete information.

Looking at the chi-squared value for this attempted reconstruction, we get a value of 1199.1066862151417. This tells us that the image is not reconstructed well at all.

4.3 Final MCMC results

In actuality, the result we see is that the reconstructed image is not like the image seen without the SN light. By just taking a look at the observed and reconstructed images side-by-side it's arguable that they look similar, but when the residual plot is taken into account, it's clear to see that the plots are in fact not similar at all.

5 Discussion of findings from MCMC

What we see in the residual plot is that there is a huge discrepancy between the observed image and reconstructed image, signifying that the image was indeed not reconstructed properly, as is expected. This is a good outcome for one main reason. It means that there is no degeneracy between the supernova light and any of the other parameters in the lens model.

This means that it is not possible for lenstronomy to replicate an image without the source light. If that actually were possible, then it would be nearly impossible to determine what is causing the reconstructed source light, whether it was left out of the program or not.

6 Conclusion

The two aims of this investigation were to produce a simulation of the evolution of a Type Ia supernova as it was being strongly gravitationally lensed, and to investigate any potentially degenerate parameters, specifically supernova source light.

The aims of this project were successful, as discussed and shown in (2.6) we successfully simulated the evolution of gravitationally lensed supernova with data and information from literature, as well as some hand picked values where necessary. Using MCMC methods in (4.3) we also successfully showed that there was no degeneracy between the supernova source light and any other parameters.

There is a bit of scope to adapt this project, the easiest would be to change the type of supernova being observed; changing the luminosity, and duration primarily. The galaxy can also be changed, the mass, ellipticity, and luminosity are easy to manipulate.

Potential further work that could be done with this project would be to investigate more than one parameter for potential degeneracy, rather than just the supernova light. It would also be fascinating to investigate more lensing configurations that are not simply an Einstein ring; generating simulated images like those would allow for the simulation of what could be observed in a more “exotic” lensing configuration.

7 Appendix.

7.1 Issues Encountered.

There were a few issues with the reconstruction of the lens image, albeit mainly due to human error.

The first was a misunderstanding about the 'gamma' parameter, this image was produced as a result of inadvertently making the lensing galaxy (significantly) denser on the outside than on the inside, which is an nonphysical result, so although the parameters converged, and there was no error with the coding, it was not a result one would ever observe due to the way it had been simulated. We can clearly see this in the convergence plot and the magnification model plot.

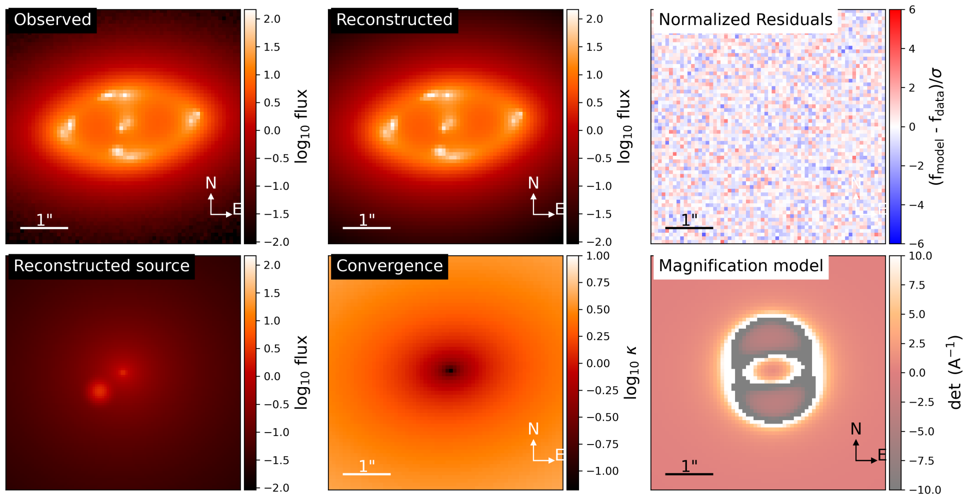

Figure 10: Observed and incorrectly reconstructed supernova image with the unusual mass distribution.

Another was due to a misunderstanding about how the parameters worked. The simulation was producing extreme residuals (meaning the image was not reconstructed properly), this was because when the fixed parameters were being played around with, they were set outside the range of what the MCMC was exploring, so the image could never reconstruct correctly.

The last main issue with MCMC was the reconstruction of the source. This was due to the way the parameters were accessed in the MCMC. This essentially boiled down to confusion with numbers, some of the wrong key word arguments were being accessed, or some were being redefined partway through the code. To circumvent these issues, in the primary python file for generating the plot with all the parameter for the lens, galaxy, and supernova; they were exported to .csv files. Then in the MCMC Python file, the program would read in from those .csv files. This meant that no issues could be made in terms of declaring variables, and the values would always be correct.

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